Trust the Units

Most of us are familiar with canceling common factors:

$$\frac{12}{10}$$

$$\frac{\cancel{2} * 2 * 3}{\cancel{2} * 5}$$

$$\frac{6}{5}$$

Maybe you are also familiar with canceling variables:

$$\frac{12x^2}{10x}$$

$$\frac{\cancel{2} * 2 * 3 * \cancel{x} * x}{\cancel{2} * 5 * \cancel{x}}$$

$$\frac{6x}{5}$$

Did you know that units behave just like variables in mathematical expressions?

$$\frac{12x^2 * hours}{10x * hours}$$

$$\frac{\cancel{2} * 2 * 3 * \cancel{x} * x * \cancel{hours}}{\cancel{2} * 5 * \cancel{x} * \cancel{hours}}$$

$$\frac{6x}{5}$$

Sometimes when we are solving mathematical problems, we get tired of carrying the units around like they are just extra baggage, but they can help keep us on track.

What is the area of a room that is 3 meters by 4 meters?

It is very tempting (and not unreasonable in such a trivial problem) to answer:

$$3x4$$

$$12$$

Twelve what? It's area, so it would be square meters or 12 m^2.

But what if we included the units when solving? Then the resulting units are in our answer!

$$3m * 4m$$

$$12 * m * m$$

$$12\ m^2$$

Trust the Units

Likewise, units cancel.

If the area of a room is 20 m^2 and one side is 4 meters, what is the length of the other side?

$$\frac{20m^2}{4m}$$

$$\frac{\cancel{2*2*}5\cancel{*m}*m}{\cancel{2*2*m}}$$

$$5m$$

These are very simple examples, but it is best to practice with the simple to gain confidence when given more complex examples.

How many 6 mg supplement capsules should my son take if the recommended dosage is 0.5 mg/kg and he weighs 200 pounds.

First, we need to convert pounds to kilograms. The conversion rate is 0.45 kg per pound or 2.2 pounds per kilogram. It doesn't matter which rate you use since they are equivalent, but it does matter which way you set them up, and units can help us.

$$x\ kilograms = 200\ pounds * ?$$

We want our answer to be kilograms which means we want the pounds to cancel out, so the pounds unit needs to be on the bottom, regardless of which conversion rate we use. The examples below use more precise conversion rates so that rounding issues don't get in the way of showing that these are equivalent conversions.

$x\ kg = \frac{200\ \cancel{pounds}}{1} * \frac{0.4536kg}{1\ \cancel{pound}}$	$x\ kilograms = \frac{200\ \cancel{pounds}}{1} * \frac{1\ kg}{2.204\ \cancel{pounds}}$
$x\ kg = 90.7kg$	$x\ kg = 90.7kg$

Next, we need to figure out how many milligrams are recommended for my 91kg son (the supplementation doesn't need to be super-precise, so I'm rounding to the nearest kg).

$$x\ mg = 91kg * ?$$

Now we want the kilograms to cancel out, and it turns out our supplement recommendation already has the kilograms on the bottom. But, if the recommendation had been "for every 1 kg, give 0.5 mg" (i.e., 1kg/0.5mg), we would just invert the rate before using.

$$x\ kg = \frac{91\cancel{kg}}{1} * \frac{0.5mg}{\cancel{kg}}$$

$$x\ kg = 45mg$$

Almost there! The last step is to figure out how many capsules we need given 6mg/capsule. This time we want milligrams to cancel out, so we invert the conversion rate.

$$x\ capsules = \frac{45\cancel{mg}}{1} * \frac{1\ capsule}{6\cancel{mg}} = 7.5\ capsules$$

Trust the Units

With units to guide us, we could also just multiple 200 pounds by each conversion rate, trusting our units to keep track of whether we need to invert the conversion rate, and even to make sure that we have correctly converted the value. For example, if you forget to include the recommended dosage in your equation, the units in your answer would be kilograms * capsules instead of capsules. If you just use numbers without units, it is very easy to miss a conversion rate or to use the inverse of the rate that you need.

So, don't think of units as unnecessary baggage, but as trusty friends that keep us from getting lost in the numbers.

$$x \text{ capsules} = \text{pounds} * \text{kilograms-per-pound-conversion}$$

$$* \text{ milligrams-per-kilograms-dose} * \text{capsule-has-milligrams-supplement}$$

$$x\ cap. = \frac{200\ \cancel{lbs}}{1} * \frac{0.45\ \cancel{kg}}{1\ \cancel{lb}} * \frac{0.5\cancel{mg}}{\cancel{kg}} * \frac{1\ cap.}{6\cancel{mg}} = 7.5\ cap.$$