Rational numbers as decimals and as integer fractions

Given a rational number expressed as an integer fraction **reduced to the lowest terms**, the quotient of that fraction will be:

- an **integer**, if the denominator is 1

- a **terminating decimal**, if the prime factors of the denominator are only powers of 2 and/or 5

- a **repeating decimal**, if the two previous requirements are not met; note that the repetend (the repeating part) may have up to [denominator minus 1] digits (e.g. if the denominator is 13, the repetend can be up to 12 digits long).

A given terminating decimal can be converted to an integer fraction very easily. The denominator will be 10 to the power of [the number of digits in the decimal], and the numerator will be the decimal multiplied by the denominator. For example, 0.42 has 2 digits in the decimal, so the denominator will be 10^2 or 100 and the numerator will be $0.42 \cdot 10^2$ or 42 . So the equivalent fraction will be $\dfrac{42}{100}$.

In addition, every repeating decimal can be systematically converted to an integer fraction, but the explanation is more involved and is given below in the section titled "Converting repeating decimals fractions to integer fractions".

Reducing integer fractions

Integer fractions can be reduced to the lowest terms by factoring the numerator and denominator and then canceling all factors that are common to both parts. However, the process for converting repeating decimals to integer fractions often results in fractions with large numerators and denominators, which are difficult to factor. For such fractions, it may be easier to use the Euclidean algorithm for finding the Greatest Common Factor (GCF) of two integers. The GCF, which is the product of all the common factors of two or more integers, is also known as the Greatest Common Divisor (GCD).

To find the GCF using the Euclidean algorithm, divide the larger number by the smaller number. The remainder will be the new smaller number and the original smaller number will be the new larger number. Repeat this process until the remainder is zero. The last remainder before the zero remainder is the GCF.

Note: if the last remainder before 0 is one, then the original integer fraction is already expressed in the lowest terms.

For example, given the fraction $\dfrac{428571}{9999990}$, we can find the GCF as shown in the chart below:

larger number	divided by smaller number	quotient	remainder
9999990	428571	23	142857
428571	142857	3	0

Dividing both the numerator and the denominator by 142,857 reduces the original fraction to $\dfrac{3}{70}$.

To simplify the process, consider showing the division as a running calculation, dividing the previous number by the new remainder until the remainder is 0:

quotient	remainder
	9999990
	428571
23	142857
3	0

Converting repeating decimals fractions to integer fractions

One method for converting a repeating decimal into an integer fraction is to use an algebraic approach as shown in the following example:

First, equate a variable to the given repeating decimal.

$$x = 0.0\overline{428571}$$

Then multiply both sides of the equation by 10 to the power of [the number of digits in the repetend]. In this example, the repetend is $428,571$, so multiply both sides of the equation by 10^6.

$$1,000,000x = 42,857.\overline{1428571}$$

Next, subtract x from both sides of the equation.

$$
\begin{array}{r}
1,000,000x = 42,587.\overline{1428571} \\
- x = 0.0\overline{428571} \\
\hline
999,999x = 42,857.1
\end{array}
$$

Now get rid of the fractional part by multiplying both sides of the equation by 10 to the power of [the number of digits in the decimal part]. In this example, the decimal part is $.1$, so multiply both sides of the equation by 10^1.

$$9,999,990x = 428,571$$

Finally, divide both sides of the equation by the coefficient of x.

$$x = \frac{428,571}{9,999,990}$$

The resulting fraction can then be reduced to $\dfrac{3}{70}$ using the Euclidean algorithm shown above.

Note: if there is any whole part in the original decimal, then the result will be a mixed number. Just calculate the fraction for the decimal part and then add the whole amount for the final answer. For example, $17.0\overline{428571}$ would reduce to $17\dfrac{3}{70}$.

A shortcut method

Based on the algebraic method for converting repeating decimals to integer fractions, we can use the following shortcut method.

Subtract the non-repeating part (without decimals) from the non-repeating part followed by one repetition of the repetend (without decimals). Using the same example, $0.0\overline{428571}$, the calculation would be $428571 - 0$. This is the numerator.

Append one 9 for each digit in the repetend followed by one 0 for each digit between the decimal point and the first digit of the repetend. For this example, that would be six 9's followed by one 0, or $9,999,990$. This is the denominator. This shortcut will result in the same integer fraction as the longer algebraic method:

$$\frac{428,571}{9,999,990}$$

As before, the resulting fraction can then be reduced to $\dfrac{3}{70}$ using the Euclidean algorithm shown above.

More examples

Convert

repeating decimal	$0.\overline{714285}$
non-repeating part	(none)
repetend	714285
numerator	714285-(nothing)=714285
denominator	999999

Find the GCF

quotient	remainder
	999999
	714285
1	285714
2	142857
2	0

Reduce

$$\frac{714285/142857}{999999/142857} = \frac{5}{7}$$

Convert

repeating decimal	$0.023\overline{428571}$
non-repeating part	023
repetend	428571
numerator	023425871-023 = 23428548
denominator	999999000

Find the GCF

quotient	remainder
	999999000
	23428548
42	15999984
1	7428564
2	1142856
6	571428
2	0

Reduce

$$\frac{23428548/571428}{999999000/571428} = \frac{41}{1750}$$

Convert

repeating decimal	$0.4\overline{72}$
non-repeating part	4
repetend	72
numerator	472-4=468
denominator	990

Find the GCF

quotient	remainder
	990
	468
2	54
8	36
1	18
2	0

Reduce

$$\frac{468/18}{990/18} = \frac{26}{55}$$

Convert

repeating decimal	$0.00\overline{5}$
non-repeating part	0
repetend	05
numerator	005-0=5
denominator	990

Find the GCF

5 is prime and 990 is divisible by 5, so 5 is the GCF

Reduce

$$\frac{5/5}{990/5} = \frac{1}{198}$$