Direct and Inverse Proportional Relationships Simplified

Introduction

A ratio compares two numbers (e.g., 12:11, 0.39:1, 15:4).

A rate is a ratio that compares two numbers that have different units (e.g.,12 boys to 11 girls in a class, 0.39 inches per centimeter, $15 for 4 yards of material, etc.). To manipulate these mathematically, we often represent them as fractions. For example:

$$\frac{12\ boys}{11\ girls} \qquad \frac{0.39\ in.}{1\ cm} \qquad \frac{\$15.00}{4\ yards}$$

Note that in these examples, the "1" in "1 *cm*" and "1 *yard*" is explicitly shown for demonstration, but it is usually not shown as it is implied.

Rates are "like" if they compare the same kinds of things/units (e.g., two rates comparing boys to girls, inches to centimeters, or cost to length of material).

Rates have a direct proportional relationship if an increase in one unit causes an increase in the other unit by the same rate of change. For example, if gas is $3 per gallon, then if you double the number of gallons, you will double the cost.

Rates have an inverse proportional relationship if an increase in one unit causes a decrease in the other unit by the same rate of change. For example, if it takes 1 person 4 hours to load a truck, then if you double the people, you will halve the time. Of course, examples don't always work out in the real world exactly like the mathematical models predict, but they are still very useful.

If two rates have a direct or indirect proportional relationship, we can use that to find missing information.

Find missing information in a direct proportion

To find a missing piece of information in a direct proportion, first write the two rates, being sure to use the same kinds of things in the numerators (top) and the same kinds of things in the denominators (bottom). Then put an equal sign in the middle and use cross multiplication (multiply the numerator of the rate on the left by the denominator of the rate on the right and the denominator of the rate on the left by the numerator of the rate on the right). Finally, divide both side by the part that will leave your missing part "alone" due to cancelation and solve

$$\frac{a}{b} = \frac{c}{d}$$

$$a * d = b * c$$

$$\frac{a * \not{d}}{\not{d}} = \frac{b * c}{d}$$

$$a = \frac{b * c}{d}$$

There are other algebraic steps that you could take to reach the same conclusion — the process shown here is to give you a simple, consistent method.

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Find missing information in an inverse proportion

The primary motive for writing this paper is that I found that the steps used in solving inverse proportions to be overly complicated and confusing to students. Sometimes the rates would be set up one way and sometimes another. This method gets you to using the same method that you use for direct proportions in one easy step.

<table>
<tr><td>My car gets 30mpg (miles per gallon). How many gallons of gas will I use if I travel 100 miles?</td><td>Our usual order of hay will feed 5 cows for 8 days. How many days will it last if I must feed 12 cows?</td></tr>
<tr><td colspan="2">As usual, write the two rates, being sure to use the same kinds of things in the numerators (top) and the same kinds of things in the denominators (bottom).</td></tr>
<tr><td>$\frac{30\ mi.}{1\ gal.}\ \frac{100\ mi.}{x\ gal.}$</td><td>$\frac{5\ cows}{8\ days}\ \frac{15\ cows}{x\ days}$</td></tr>
<tr><td colspan="2">Ask yourself whether this is a direct or inverse proportion.</td></tr>
<tr><td>As the distance I drive increases, my car’s fuel consumption will also increase, so this is a direct proportion.</td><td>As the number of cows increases, the days that the feed will last will decrease, so this is an inverse proportion.</td></tr>
<tr><td>If the proportion is a direct proportion, put an equal sign in the middle and use cross multiplication followed by division to solve.</td><td>If it is an inverse proportion, put an I (for Inverse) in the middle. Then swap the numerators, put an equal sign in the middle and use cross multiplication followed by division to solve.
Note that you could swap the denominators instead of the numerators, but the main thing is to be consistent with your process.</td></tr>
<tr><td></td><td>$\frac{5\ cows}{8\ days}\ I\ \frac{15\ cows}{x\ days}$</td></tr>
<tr><td>$\frac{30\ mi.}{1\ gal.} = \frac{100\ mi.}{x\ gal}$</td><td>$\frac{15\ cows}{8\ days} = \frac{5\ cows}{x\ days}$</td></tr>
<tr><td>$30\ mi. * x\ gal. = 100\ mi. * 1\ gal.$</td><td>$15\ cows * x * days = 5\ cows * 8\ days$</td></tr>
<tr><td>$x = \frac{100\ \cancel{mi} * 1\ \cancel{gal}}{30\ \cancel{mi * gal.}}$</td><td>$x = \frac{\cancel{5\ cows} * 8\ \cancel{days}}{3 * \cancel{5\ cows * days}}$</td></tr>
<tr><td>$x = 3\ \frac{1}{3}$</td><td>$x = 2\ \frac{2}{3}$</td></tr>
</table>

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Caution: make sure that you are comparing like things (trust your units)

Our co-op has 3 girls to every 5 boys. There are 80 students in total. How many are girls?

If we get tired of carrying our units around, we might be tempted to write:

$$\frac{3}{5} = \frac{x}{80}$$

But when we include our units, we can see right away that these rates are not “like”:

$$\frac{3\ girls}{5\ boys} = \frac{x\ girls}{80\ students}$$

So, we need to make them “like”: 3 girls plus 5 boys is 8 students

$$\frac{3\ girls}{8\ students} = \frac{x\ girls}{80\ students}$$

$$8\ students * x\ girls = 3\ girls * 80\ students$$

$$x\ = \frac{3\ \cancel{girls} * 8 * \cancel{10\ students}}{\cancel{8\ students * girls}}$$

$$x = 24$$

Another common “gotcha” with unlike rates is mixing minutes and hours.

If I walk 2 mph, how far can a get in 10 minutes?

Not like:

$$\frac{2\ miles}{hour} = \frac{x\ miles}{10\ minutes}$$

Like:

$$\frac{2\ miles}{60\ minutes} = \frac{x\ miles}{10\ minutes}$$

Or:

$$\frac{2\ miles}{hour} = \frac{x\ miles}{\frac{1}{6}hour}$$