Use of Absolute Value When Simplifying Radical Expressions

Given:

- *a* is a real number
- the radical index *n* is an integer $\geq$ 2 (i.e., {2, 3, 4, …})
- *power* is a positive integer (i.e., {1, 2, 3, ...})
- *oddpower* is a positive odd integer (i.e., {1, 3, 5, ...})
- *evenpower* is a non-negative even integer (i.e., {0, 2, 4, ...})

In simplifying radical expressions, you only need to use absolute value to ensure that the expression cannot yield an invalid result **if the radical index *n* is even** (i.e., {2, 4, 6, …}) and:

$$\sqrt[n]{a^{power}}$$

can be rewritten as:

$$\sqrt[n]{(a^{oddpower})^n \cdot (a^{evenpower})}$$

such that *evenpower* is less than the radical index *n* so that taking the root of $(a^{oddpower})^n$ will fully simplify the radical.

The above expression simplifies to:

$$\left|a^{oddpower}\right| \cdot \sqrt[n]{a^{evenpower}}$$

Note that this includes the special case:

$$\sqrt[n]{(a^{oddpower})^n \cdot (a^0)}$$

which simplifies to:

$$\left|a^{oddpower}\right|$$

See the following page (or your textbook) for further explanation, but when you are simplifying radicals, you just need to check for the case described above.

On a related note, when you are simplifying fractions with radicals in the numerator and denominator, it is best to reduce the fractions under the radicals in the numerator and denominator before simplifying the radicals and rationalizing the denominator (otherwise, you may miss some required absolute values).

Why is absolute value needed in the above case?

Absolute value is needed because the expression $a^{oddpower} \cdot \sqrt[n]{a^{evenpower}}$ could yield a negative result which would not be valid for the original expression.

What about the other cases?

- Absolute value is not needed if the radical index is odd (i.e., $\{3, 5, 7, \dots\}$) because the root can be negative (i.e., a negative result for the expression is valid).

- Absolute value is not needed if the radical index is even but the remaining power under the radical after simplification is odd, because you can't take an even root of a negative number (i.e., it would be invalid to substitute a negative number for *a*).

$$a^{oddpower} \cdot \sqrt[n]{a^{oddpower}} \qquad or \qquad a^{evenpower} \cdot \sqrt[n]{a^{oddower}}$$

- Absolute value is not needed if the radical index is even and the both the power outside the radical and the power under the radical after simplification are even, because the expression the expression cannot yield a negative (and therefore invalid) result.

$$a^{evenpower} \cdot \sqrt[n]{a^{evenpower}}$$